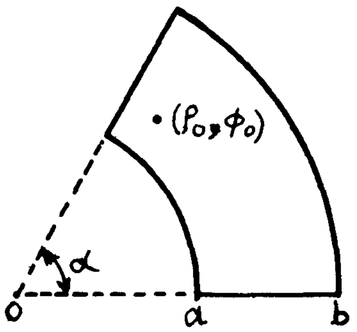


Fig. 6. An annular sector.

The angle of the sector is $\alpha = \pi/l$, and $n_i = nl$. The function $F_{mn_i}(\rho)$ is given by (15) and k_{mn_i} are obtained by solving (17). It can be seen that this Green's function is consistent with the Green's function for the annular ring in the same way as the Green's function for a circular sector is with that of a circle.

Using the Green's functions given by (8), (14), and (18), the Green's function technique of analyzing planar circuits and microstrip antennas can be extended to circuits incorporating circular sectors, annular rings and annular sectors. These Green's functions are valid both for triplate stripline type circuits and for open microstrip type circuits.

References

[1] T. Okoshi and T. Miyoshi, "The planar circuit—An approach to microwave integrated circuitry," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 245–252, Apr. 1972.

[2] Y. T. Lo, D. Solomon, and W. F. Richards, "Theory and experiment on microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 137–145, Mar. 1979.

[3] T. Okoshi, T. Takeuchi, and J. P. Hsu, "Planar 3-db Hybrid Circuit," *Electron. Commun. Japan*, vol. 58-B, no. 8, pp. 80–90, Aug. 1975.

[4] R. Chadha and K. C. Gupta, "Green's functions for triangular segments in planar microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1139–1143, Oct. 1980.

[5] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*. New York: McGraw-Hill, 1953, ch. 7, p. 820.

Segmentation Method Using Impedance Matrices for Analysis of Planar Microwave Circuits

RAKESH CHADHA, STUDENT MEMBER, IEEE, AND K. C. GUPTA, SENIOR MEMBER, IEEE

***Abstract*—Segmentation method for the analysis of two-dimensional microwave planar circuits is modified by using Z-matrices for the individual planar segments. The proposed method is compared with the previously reported method using S-matrices and is shown to be computationally more efficient.**

Manuscript received June 9, 1980; revised September 2, 1980. This work was supported by the Department of Electronics, Government of India.

The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Kanpur 208016, India.

I. Introduction

Two-dimensional microwave planar circuits have been proposed for use in microwave integrated circuits [1]. One of the methods, for analyzing planar components, involves determination of Z-matrix of the component using Green's functions. The Green's functions are available for only a few regular shapes. Analysis of other shapes is normally done by segmenting these into shapes for which the Green's functions are known. The Segmentation Method [2], [3] uses this approach and combines the S-matrices of the individual components to obtain overall S-matrix. A formulation for combining segments to form 2-port and 4-port circuits is given in [2], [3] and could be extended to any general n-port network.

In this method a considerable effort is spent in computing S-matrices for each of the segments. These matrices are then combined to obtain the overall S-matrix. Considerable reduction in computational effort can be achieved if Z-matrices of individual components are combined to give the overall Z-matrix from which the overall network scattering matrix may be determined. A segmentation method that combines Z-matrices of the segments is described in this article.

II. Segmentation Using S-Matrices

In the segmentation method using S-matrices, we proceed as follows. The S-matrices for the individual segments are put together as [4]

$$\begin{bmatrix} \bar{b}_p \\ \bar{b}_c \end{bmatrix} = \begin{bmatrix} \tilde{S}_{pp} & \tilde{S}_{pc} \\ \tilde{S}_{cp} & \tilde{S}_{cc} \end{bmatrix} \begin{bmatrix} \bar{a}_p \\ \bar{a}_c \end{bmatrix} \tag{1}$$

where $\bar{a}_p$ $\bar{b}_p$ and $\bar{a}_c$, $\bar{b}_c$ are the normalized wave variables at the p external and c internal connected ports. The interconnection constraints are given as

$$\bar{b}_c = \tilde{\Gamma} \bar{a}_c \tag{2}$$

where $\tilde{\Gamma}$ is the connection matrix. The overall S-matrix is obtained as

$$\tilde{S}_p = \tilde{S}_{pp} + \tilde{S}_{pc}\left(\tilde{\Gamma} - \tilde{S}_{cc}\right)^{-1} \tilde{S}_{cp}. \tag{3}$$

The solution of (3) requires inversion of a matrix of order equal to the number of interconnected ports. Let us consider an example shown in Fig. 1 to illustrate the total computational effort needed in the segmentation method. This network is a planar circuit version of a compensated in-line power divider [5]. Segment B, and C_1 and C_2 parts of segment C are quarter-wave transformers with characteristic impedances equal to $Z_0/(2^{1/4})$ and $(2^{1/4})Z_0$, respectively. Segments A, D, and E are portions of outgoing transmission lines (characteristic impedance Z_0). These three segments are considered as planar components in order to take into account any higher order modes that may be excited by the discontinuities present at the ends of three transformers. For better accuracy, each external port is divided into six subports for obtaining the Z-matrix. The six subports are combined together using ideal six-way power dividers (not shown in the figure) at each external port [3]. To obtain S-matrices of individual segments, five complex matrices, three of order 12 and one each of order 14 and 20, are to be inverted. The number of interconnections in the network is now 44 and so a complex matrix of order 88 has to be inverted to obtain the overall scattering matrix.

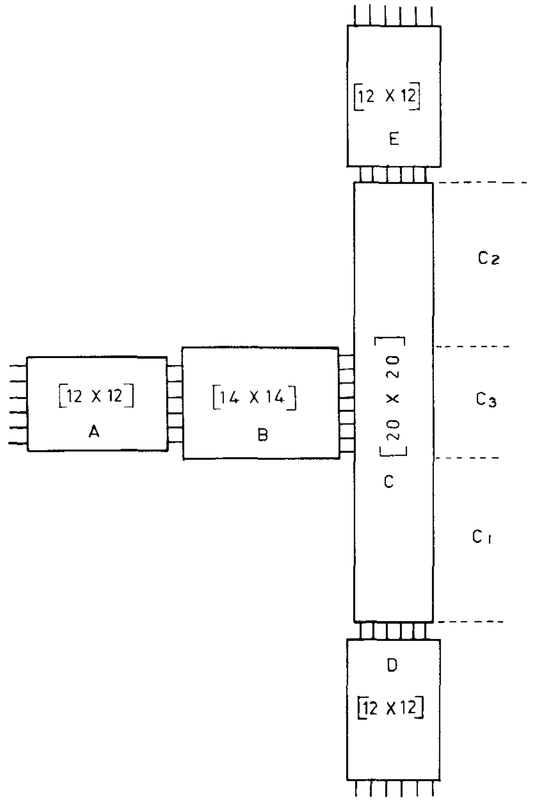


Fig. 1. A typical planar circuit analyzed by segmentation method.

It has been pointed out by Monaco and Tiberio [6], that with suitable ordering of rows and columns it is possible to write the connection matrix in the form

$$\tilde{\Gamma}=\begin{bmatrix} \tilde{0} & \tilde{I} \\ \tilde{I} & \tilde{0} \end{bmatrix} \tag{4}$$

where $\tilde{I}$ is an identity matrix and $\tilde{0}$ a null matrix, each of order $c/2$. They have also pointed out that when two segments are being interconnected, submatrix $\tilde{S}_{cc}$ in (1) can be written in the block diagonal form as

$$\tilde{S}_{cc}=\begin{bmatrix} \tilde{M} & \tilde{0} \\ \tilde{0} & \tilde{N} \end{bmatrix} \tag{5}$$

where $\tilde{M}$ and $\tilde{N}$ are $c/2 \times c/2$ matrices. In such cases $(\tilde{\Gamma}-\tilde{S}_{cc})$ becomes of the form

$$(\tilde{\Gamma}-\tilde{S}_{cc})=\begin{bmatrix} -\tilde{M} & \tilde{I} \\ \tilde{I} & -\tilde{N} \end{bmatrix}. \tag{6}$$

The inversion of $(\tilde{\Gamma}-\tilde{S}_{cc})$ in the above form requires inverse of two matrices of order $c/2$ and thus results in saving of computational effort [6].

This technique can be extended for interconnection of more than two segments also if it is possible to write $\tilde{S}_{cc}$ in block diagonal form of (5). This is possible if the network does not contain any loop with odd number of segments in it. For example, it is possible to write $\tilde{S}_{cc}$ in block diagonal form for the circuit shown in Fig. 1. Combination of S-matrices would now require inversion of two complex matrices of order 44 each.

III. Segmentation Using Z-Matrices

The computational efficiency of the segmentation method discussed in Section II can be improved if the Z-matrices of individual segments are combined to give the overall Z-matrix. It may be recalled that for lossless networks, Z-matrices are purely imaginary. Multiplication and inversion of purely imaginary matrices can be carried out with the same computational effort as required for real matrices. In a general network of segments, the Z-matrices can be written together as

$$\begin{bmatrix} \bar{V}_p \\ \bar{V}_c \end{bmatrix}=\begin{bmatrix} \tilde{Z}_{pp} & \tilde{Z}_{pc} \\ \tilde{Z}_{cp} & \tilde{Z}_{cc} \end{bmatrix}\begin{bmatrix} \bar{I}_p \\ \bar{I}_c \end{bmatrix} \tag{7}$$

where $\bar{V}_p$, $\bar{I}_p$ and $\bar{V}_c$, $\bar{I}_c$ are the voltages and currents at the p external and c internal connected ports. The interconnection constraints, that the voltages at the two connected ports are equal and the sum of currents at the two connected ports is zero, can be expressed as

$$\tilde{\Gamma}_1\bar{V}_c=\bar{0} \tag{8a}$$

$$\tilde{\Gamma}_2\bar{I}_c=\bar{0} \tag{8b}$$

where $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ are two matrices with $c/2$ rows and c columns describing the topology. In these matrices each row describes a connection such that all entries in a row are zero except the two in the columns corresponding to the two connected ports. The two nonzero entries in a row are 1 and -1 for matrix $\tilde{\Gamma}_1$ and both 1 for matrix $\tilde{\Gamma}_2$.

A relation between $\bar{I}_c$ and $\bar{I}_p$ is obtained using (7) and (8a). This on combining with (8b) becomes

$$\begin{bmatrix} \tilde{\Gamma}_1\tilde{Z}_{cc} \\ j\tilde{\Gamma}_2 \end{bmatrix}\bar{I}_c=\begin{bmatrix} -\tilde{\Gamma}_1\tilde{Z}_{cp} \\ \tilde{0} \end{bmatrix}\bar{I}_p \tag{9}$$

where $\tilde{0}$ is a $(c/2)\times p$ null matrix and the left-hand side of (8b) has been multiplied by j so that the matrix on the left-hand side of (9) becomes purely imaginary for lossless networks. Substituting the value of $\bar{I}_c$, obtained from (9) into (7), we get the overall network impedance matrix as

$$\tilde{Z}_p=\tilde{Z}_{pp}-\tilde{Z}_{pc}\begin{bmatrix} \tilde{\Gamma}_1\tilde{Z}_{cc} \\ j\tilde{\Gamma}_2 \end{bmatrix}^{-1}\begin{bmatrix} \tilde{\Gamma}_1\tilde{Z}_{cp} \\ \tilde{0} \end{bmatrix}. \tag{10}$$

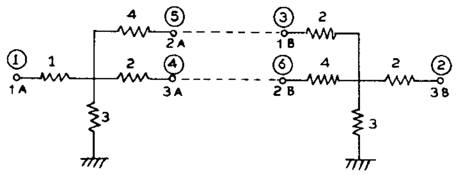
This requires the inversion of a matrix of order equal to the number of interconnected ports.

The multiple subports at each external port are combined as follows. It is assumed that only the TEM mode is present in the uniform transmission line at the location of the external port. This is a valid assumption if the external ports are taken at a certain distance away from the planar circuit since any higher order modes present at the discontinuity at the junction of the transmission line and the planar circuit would decay along the uniform transmission line. For considering the external ports at a distance away from the discontinuities, the intermediate transmission lines are modelled as rectangular planar segments (as shown in Fig. 1 for the power-divider circuit).

Since only TEM mode is assumed to be present at an external port, the voltages at the subports of an external port are same. Thus, one can make use of the parallel connection of subports which implies that the total current injected in a port gets divided into its various subports. This combination requires inversion of the overall Z-matrix to obtain admittance matrix. In general, if ports I and J are divided into subports $I=\{i_1, i_2, \cdots\}$ and $J=\{j_1, j_2, \cdots\}$, then the term Y_{IJ} of the overall admittance matrix is given as

$$Y_{IJ}=\sum_{k\in I}\sum_{l\in J} y_{kl} \tag{11}$$

where y_{kl} are the terms of the admittance matrix with multiple



$$\tilde{Z}_A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 7 & 3 \\ 3 & 3 & 5 \end{bmatrix} ; \quad \tilde{Z}_B = \begin{bmatrix} 5 & 3 & 3 \\ 3 & 7 & 3 \\ 3 & 3 & 5 \end{bmatrix}$$

Fig. 2. A lumped network example for illustrating the proposed method.

subports. The overall scattering matrix may be obtained from the admittance matrix.

If the external ports are located such that the TEM mode approximation is not valid, the voltage and current density over the width of an external port can be expanded in terms of higher order stripline modes. The higher order stripline modes decay away from the junction along the transmission line and their characteristic impedances are reactive. Therefore, these higher order modes are considered as terminated by these reactive impedances at the port location itself [7]. However, for purposes of characterization of the planar circuit, it is convenient that the external ports be chosen at the locations where only the dominant mode exists, so that the procedure described earlier is applicable.

For analyzing the circuit shown in Fig. 1 by the method described above, two real matrices of order 52 and 18 and a complex matrix of order 3 need be inverted. It is seen that there is considerable saving in computational effort by using this method.

Further saving in computational effort can be achieved if the connected ports are suitably subgrouped and Z-matrix formulation used as follows. The c connected ports are divided into groups q and r, each containing $c/2$ ports. This is done in such a way that q_1 and r_1 ports are connected together, q_2 and r_2 ports are connected together and so on. This involves reordering of rows and/or columns for $\tilde{Z}_{cp}$, $\tilde{Z}_{pc}$, and $\tilde{Z}_{cc}$ as given in (7). The Z-matrices can now be written together as

$$\begin{bmatrix} \bar{V}_p \\ \bar{V}_q \\ \bar{V}_r \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{pp} & \tilde{Z}_{pq} & \tilde{Z}_{pr} \\ \tilde{Z}_{qp} & \tilde{Z}_{qq} & \tilde{Z}_{qr} \\ \tilde{Z}_{rp} & \tilde{Z}_{rq} & \tilde{Z}_{rr} \end{bmatrix} \begin{bmatrix} \bar{I}_p \\ \bar{I}_q \\ \bar{I}_r \end{bmatrix}. \tag{12}$$

In this case the interconnections can be expressed in a much simpler form as

$$\bar{V}_q = \bar{V}_r \tag{13a}$$

$$\bar{I}_q + \bar{I}_r = 0. \tag{13b}$$

Substituting (13) in (12) and eliminating $\bar{V}_q$, $\bar{V}_r$, $\bar{I}_q$, and $\bar{I}_r$, the Z-matrix of the overall network is given by

$$\tilde{Z}_p = \tilde{Z}_{pp} + (\tilde{Z}_{pq} - \tilde{Z}_{pr})(\tilde{Z}_{qq} - \tilde{Z}_{qr} - \tilde{Z}_{rq} + \tilde{Z}_{rr})^{-1} \cdot (\tilde{Z}_{rp} - \tilde{Z}_{qp}). \tag{14}$$

The method described above is illustrated by combining two 3-port resistive subnetworks shown in Fig. 2, to yield an overall 2-port network. The impedance matrices of the two subnetworks are also given in Fig. 2. For combining the subnetworks using the method described above, the ports of the two subnetworks are numbered (port numbers encircled) as shown in Fig. 2. The Z-matrices can be written together as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 3 & 3 & 0 \\ 0 & 5 & 3 & 0 & 0 & 3 \\ 0 & 3 & 5 & 0 & 0 & 3 \\ 3 & 0 & 0 & 5 & 3 & 0 \\ 3 & 0 & 0 & 3 & 7 & 0 \\ 0 & 3 & 3 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{15}$$

and the overall Z-matrix is given by (14), as

$$\tilde{Z}_p = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} 12 & -6 \\ -6 & 12 \end{bmatrix}^{-1} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} \tag{16}$$

which can be seen to be the Z-matrix of the combination.

The method, using subgrouped interconnections, as given by (14) and illustrated in the example presented above, requires the inversion of a matrix of order equal to the number of interconnections, i.e., $c/2$. For analyzing the circuit shown in Fig. 1, we need to invert two real matrices of orders 26 and 18 and a complex matrix of order 3.

TABLE I
ORDERS OF MATRICES TO BE INVERTED IN VARIOUS METHODS OF SEGMENTATION FOR THE CIRCUIT OF FIG. 1

Method	Segmentation using S-matrices	Segmentation using Z-matrices
Without subgrouping the connected ports	20×20 complex 3×(12×12) complex 14×14 complex 88×88 complex	52×52 real 18×18 real 3×3 complex
With suitable subgrouping of the connected ports	20×20 complex 3×(12×12) complex 14×14 complex 2×(44×44) complex	26×26 real 18×18 real 3×3 complex

IV. CONCLUDING REMARKS

Orders of matrices that need to be inverted in different methods are compared in Table I. It may be noted that segmentation using Z-matrices with suitably subgrouped interconnections involves significantly smaller amount of computations as compared to the other methods available earlier. Moreover, this method can be used for any arbitrary topology whereas, as pointed out in Section II, the method using S-matrices with subgrouping of connected ports can be used only when the network does not contain any loop with odd number of segments in it.

Acknowledgment

Discussions with G. Kumar and P. C. Sharma are thankfully acknowledged.

References

[1] T. Okoshi and T. Miyoshi, "The planar circuit—An approach to microwave integrated circuitry," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 245–252, Apr. 1972.
[2] T. Okoshi, Y. Uehara, and T. Takeuchi, "The segmentation method—An approach to the analysis of microwave planar circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 662–668, Oct. 1976.
[3] T. Okoshi and T. Takeuchi, "Analysis of planar circuits by segmentation method," *Electron. Commun. Japan*, vol. 58-B, no. 8, pp. 71–79, Aug. 1975.
[4] V. A. Monaco and P. Tiberio, "Computer-aided analysis of microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 249–263, Mar. 1974.
[5] H. Howe, Jr., *Stripline Circuit Design*. Dedham, MA: Artech House, 1974, ch. 3, p. 95.
[6] V. A. Monaco and P. Tiberio, "Automatic scattering matrix computation of microwave circuits," *Alta Freq.*, vol. 39, pp. 59–64, Feb. 1970.
[7] T. Miyoshi and S. Miyauchi, "The design of planar circulators for wide-band operation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 210–214, Mar. 1980.

Letters

Correction to "Dispersion Relations for Comb-Type Slow-Wave Structures"

I. L. VERBITSKII

The following corrections should be made to the original paper.[1]

On page 49, "f" in the numerator of (1) should read "b."

On page 49, column 1, two lines below (1), "det" should read "$\underset{=}{\text{def}}$" (to mean "equality by definition").

On page 49, column 1, on the left-hand side of (3), "f" should read "$-f$." A period should appear at the end of the equation. On the next line, "with" should read "With."

On page 49, column 2, five lines below (4b), "low-phase velocities" should read "low phase velocities." On the next to last line in the same paragraph, "$(2\beta h/\pi)^2 \ll 1$" should read "$(2\beta \text{h}/\pi)^2 \gg 1$."

On page 50, column 1, the first line after the first equation in the column, "$b=i/2+it$" should read "$b=1/2+it$." On the next line, "$0 \leqslant t \geqslant \infty$" should read "$0 \leqslant t \leqslant \infty$." On the last line in the same paragraph, "$\theta = I$" should read "$\theta = 1$."

Manuscript received August 18, 1980.
The author is with the Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian S.S.R., Kharkov 85, U.S.S.R.
[1] I. L. Verbitskii, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 48–50, Jan. 1980.

Correction to "A Theoretical Basis for Microwave and RF Field Effects on Excitable Cellular Membranes"

CHARLES A. CAIN, MEMBER, IEEE

In the above paper,[1] the following corrections should be made. At the top of page 145, 10 W/cm^2 should be 10^2 W/cm^2, 194 kV/m should be 19.4 kV/m, 9.8 mV should be 0.98 mV, and the duty cycle of 0.001 should be 0.0001.

Manuscript received August 27, 1980.
The author is with the University of Illinois, Urbana, IL 61801.
[1] C. A. Cain, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 142–147, Feb. 1980.